BESTPRICE

White Paper - v 1(draft)

LUIGI PODERICO, 2005

INDEX

INTRODUCTION....................1

Problem context....................1

Document outline....................2

DEFINITIONS....................3

Variables and parameters....................3

Constraints....................4

Objective functions....................4

THE MIN-COST MODEL....................5

The best solution at given QoS....................5

THE MAX-QOS SOLUTION....................6

The best solution at given cost....................6

1 INTRODUCTION

This document has the main purpose to describe the problems solved by BestPrice and other related considerations.

For each problem a mathematical model will be provided with an informal description.

1.1 PROBLEM CONTEXT

The telephone providers supports national as well as international calls. While in the national case a proprietary network can be used to satisfy the entire demand, the international calls must be, whole or partially, satisfied with other carriers.

Each carrier provides a complete *price list* for every destination reached, completed with the declared *Quality of Service* (QoS). For some destinations, the carrier can details these information for some locale area.

The cost is represented as a cost per minute plus a cost per call; while the QoS is represented with a real number ranging from 0 and 1: 0 is the lowest quality and 1 is the highest quality.

The destinations are identified with the international code eventually completed with the area code when needed. The following table show an example of price list.

Country	*Code*	*Cost per minute*	*Cost per call*	*QoS*
Afghanistan	93	134,35	8,76	0,56
Alaska	1907	85,8	5,59	0,58
Albania	355	43,7	2,55	0,68
Algeria	213	44,32	3,28	0,58

Table 1 Price List Example

The above information can be used to estimate cost and quality of offered service, but these are not sufficient for a complete calculus. An estimation or historical data on distribution calls must be used to have a well done service cost and quality calculation.

For each destination the number of calls and the total minutes of traffic is given, as the following example.

Destination	Area code	Total traffic	N. Calls
ROMANIA	***40***	***199156,52***	***66385***
BANGLADESH DHAKA	***8802***	***144019,44***	***48006***
BANGLADESH MOBILE	***8801***	***109142,08***	***363800***

Table 2 Example of distribution calls

1.2 DOCUMENT OUTLINE

A generic telephone provider has the problem to schedule the *best way* to route the outgoing calls, selecting for each destination a preferred carrier. This paper will explore the different way the schedule the route, using the operational research technologies.

After some basic definitions, two main problem classes will be discussed. The first class is related to cost minimization or, more in general, cost control; the second class is related to profit maximization.

2 DEFINITIONS

In this document we will use the following notation and definitions.

2.1 VARIABLES AND PARAMETERS

D - is the set of destinations;

V - is the set of vectors or carriers;

$x_{i,j} \in \{0,1\}, i \in D, j \in V$ - is the binary variable that is equal to 1 iff the destination *i* is covered by the vector *j*;

$X = \{x_{i,j}, i \in D, j \in V\}$ - is the set of decision variables;

$fx_{i,j} \in \{0,1\}, i \in D, j \in V$ - indicate with 1 iff the destination *i* could be covered by the vector *j*.

$FX = \{fx_{i,j}, i \in D, j \in V\}$ - is the set of *fx* parameters.

We have to note that while the set *X* is the output of the problem under discussion, the set *FX* describe the problem structure.

$CM : D \times V \rightarrow \mathbb{R}^+$ - is the cost per minute of a call to a destination using a given vector.

$CR : D \times V \rightarrow \mathbb{R}^+$ - is the cost on response of a call to a destination using a given vector.

$Q : D \times V \rightarrow [0,1]$ - is the quality of service assured by a vector to a given destination.

$PPM : D \rightarrow \mathbb{R}^+$ - is the price per minute which the provider resell the traffic to a given destination.

$PPA : D \rightarrow \mathbb{R}^+$ - is the price per answer which the provider resell the traffic to a given destination.

$PPMC : D \rightarrow \mathbb{R}^+$ - is the price per minute offered by the best provider competitors.

$PPAC : D \rightarrow \mathbb{R}^+$ - is the price per answer offered by the best provider competitors.

$OM : D \to \mathbb{R}^+$ - is the number of minute of estimated traffic to a given destination.

$OC : D \to \mathbb{Z}$ - is the number of estimated paying calls to a given destination.

$MU : D \to \mathbb{R}^+$ - is the maximum murk-up that the provider decide to have on a given destination.

2.2 CONSTRAINTS

The following constraints describe the covering structure of problem:

$$x_{i,j} \leq fx_{i,j} \quad \forall i \in D, \forall j \in V$$
$$\sum_{j \in V} x_{i,j} = 1 \quad \forall i \in D \qquad CC(X).$$

Formula 1 Covering constraints

The in-come is bounded by the maximum mark-up:

$$IC(i) = PPM(i) \cdot OM(i) + PPA(i) \cdot OC(i) \quad \forall i \in D$$
$$IC(i) \leq MU(i) \cdot COST(X, i) \quad \forall i \in D \qquad IC(X)$$

Formula 2 In-come constraints

Where:

$$COST(X, i) = \sum_{j \in V} x_{i,j} \cdot CM(i, j) \cdot OM(i) + \sum_{j \in V} x_{i,j} \cdot CM(i, j) \cdot OC(i)$$

Formula 3 Cost for a destination

is the overall cost for the destination *i* using a solution *X*.

The constraints on in-come when the best competitor is considered are:

$$ICC(i) = PPMC(i) \cdot OM(i) + PPAC(i) \cdot OC(i) \quad \forall i \in D$$
$$IC(i) \leq KIC(i) \cdot ICC(i) \quad \forall i \in D \qquad IIC(X)$$

Formula 4 In-come of the best competitor

where $KIC(i) \in \mathbb{R}^+, \forall i \in D$ are constants chosed by the reseller.

2.3 OBJECTIVE FUNCTIONS

In this work we will consider the following two objective functions in relation with minimum cost problems: the cost function and the quality function.

$$\sum_{i\in D}\sum_{j\in V} x_{i,j}\cdot CM(i,j)\cdot OM(i)+\sum_{i\in D}\sum_{j\in V} x_{i,j}\cdot CM(i,j)\cdot OC(i)=COST(X)$$

Formula 5 Cost function

$$\sum_{i\in D}\sum_{j\in V} x_{i,j}\cdot Q(i,j)\cdot OC(i)=QUALTY(X)$$

Formula 6 Quality function

The revenue is defined as:

$$REV(X)=\sum_{i\in D} IC(i)-COST(X)$$

Formula 7 Revenue function

3 THE MIN-COST MODEL

The following is the model of the min cost problem, where we want to find the solution at minimum cost where the total QoS is greater or equal to a given value $\bar{q}$.

$$\begin{cases} min & COST(X) \\ s.t. & CC(X) \\ & QUALITY(X) \geq \bar{q} \end{cases} \qquad MC(\bar{q})$$

Formula 8 Min cost model

We have to note that with $\bar{q}=0$ the model give the minimum cost that we have to pay to reach all the destination and the quality associated at that cost.

3.1 THE BEST SOLUTION AT GIVEN QOS

Given a total quality value $\bar{q}$, the solution of $MC(\bar{q})$ give the best way to route the traffic at that quality, because any other solution with a total quality $\bar{q}$ has a lesser cost.

4 THE MAX-QOS SOLUTION

The following is the model of the max QoS problem, where we want to find the solution at maximum quality of service, where the total cost is lesser or equal to a given value $\bar{c}$.

$$\begin{cases} max & QUALITY(X) \\ s.t. & CC(X) \\ & COST(X) \le \bar{c} \end{cases} \qquad MQ(\bar{c})$$

Formula 9 Max QoS model

We have to note that with $\bar{c}=\infty$ the model give the maximum quality of service and the associated cost.

4.1 THE BEST SOLUTION AT GIVEN COST

Given a cost $\bar{c}$, the solution of $MQ(\bar{c})$ give the best way to route the traffic at that cost, because any other solution that cost $\bar{c}$ has a lesser quality.

5 PROFIT MAXIMIZATION

5.1 PROFIT AND MARK-UP

The model describing the profit maximization with mark-up is:

$$\begin{cases} max & REV(X) \\ s.t. & CC(X) \\ & IC(X) \end{cases} \qquad PMU$$

Formula 10 Profit and mark-up

This model allow to discover the maximum profit after that the maximum mark-up is fixed. Note that the *IC(X)* constraints blind costs and in-comes.

5.2 PROFIT AND COMPETITORS

Considering the behavoir of some customers that are sensible to price rather than quality, a model considering the competitors prices is the following:

$$\begin{cases} max & REV(X) \\ s.t. & CC(X) \\ & ICC(X) \end{cases} \qquad PBC$$

Formula 11 Profit and competitors

5.3 GENERAL CONSIDERATIONS

The value of *COST(X)* and *QUALITY(X)* at the optimum solution gives, respectivelly, the cost and the quality related to the profit.